LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – **NOVEMBER 2013**

ST 5400 - APPLIED STOCHASTIC PROCESSES

Date : 15/11/2013 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

PART- A

Answer ALL the questions: (10 X 2 = 20)

1) Define a stochastic process. Give an example.
2) Define a process with independent increments.
3) Consider a Markov chain with states 0, 1 and transition probability matrix.

$$P = \begin{pmatrix} 1 & 0 \\ 1/2 & 1/2 \end{pmatrix}$$

Identify the transient state and the recurrent state.

4) Obtain the period of each state in the Markov chain with transition probability matrix and states 0,1

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

5) Obtain the classes in the following Markov chain with states 0, 1, 2, 3

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \end{bmatrix}$$

6) State Basic limit theorem.
7) Write the PGF of a Poisson process.
8) Define a n-step transition probability matrix.
9) If patients arrive at a clinic according to Poisson process with $\lambda = 2$. Find the probability that during a 1- minute interval, no patient arrives.
10) Define a stationary distribution.

PART –B

Answer any FIVE of the following: (5 X 8 =40)

11) The transition probability matrix of a Markov chain with states 0, 1, 2 is

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

and the initial distribution $P(X_0 = i) = 1/3$, i = 0, 1, 2.

Find (i) $P[X_1=2]$ (ii) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 =2]$.

12) State and prove Chapman-Kolmogorov equation.

13) If i $\leftrightarrow$ j and if i is recurrent, show that j is also recurrent.

14) Three boys A, B, C throwing a ball each other A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. Find the tpm. Verify whether the Markov chain is irreducible and aperiodic.

15) Show that in a Poisson process the time between two arrivals is exponential.

16) State and prove the additive property of Poisson process.

17) Show that the one dimensional random walk is recurrent when p = ½.

18) Let the sequence of independent random variables $X_1, X_2, \ldots$ be such that $P[X_i = k] = a_k$, $\Sigma a_k = 1$, $a_k \geq 0$. Show that $S_n = X_1 + X_2 + \ldots + X_n$ is a Markov chain. Obtain the tpm.

PART – C

Answer any TWO of the following: (2 X 20 = 40)

19) a) Show that state i is recurrent iff $\sum P_{ii}^{n} = \infty$

b) A gambler has Rs.2. He bets Re.1 at a time and wins Re.1 with probability ½. He stops playing if he loses Rs.2 or wins Rs.2 extra. Obtain the tpm of the related Markov chain. Obtain the classes, period of each state. (10+ 10)

20) a) State the theorem used to get the stationary distribution.

b) Verify whether all the conditions are satisfied for getting the stationary probabilities for the following Markov chain with tpm states 1, 2, 3, 4.

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1/2 & 1/8 & 1/8 & 1/4 \end{pmatrix}$$

Also obtain the stationary probabilities. (5 +15)

21) a) State the postulates of a Poisson process.

b) Obtain the expression for $P_n(t)$. (5+15)

22) a) Explain the classifications of a stochastic process.

b) Explain the inventory model as a Markov chain. (10 + 10)

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